1.

a) $\theta = 50°$

$\theta_r = ?$

$360° = 2\pi\,rad$ } : $\Rightarrow \frac{\theta_r}{2\pi\,rad} = \frac{50°}{360°}$ (6)

$50° = \theta_r$

$\theta_r = \frac{10}{36}\pi\,rad = 0{,}27\pi\,rad$; $\theta_r = 0{,}872\,rad$

b) $T = 24h = 24 \cdot 3600s = 86400s$

$\omega = ?$

$\omega = \frac{\theta}{t} = \frac{2\pi}{T} \Rightarrow \omega = \frac{2\pi}{86400s} = 7{,}2 \cdot 10^{-5}\frac{1}{s}$ (6)

$\omega = 7{,}2 \cdot 10^{-5}\frac{1}{s}$

c) $R = 6400\,km = 6{,}4 \cdot 10^{6}\,m$

$\omega = const$ / $\omega =$ (5)

$\alpha = \frac{\Delta\omega}{\Delta t} = 0$

$\vec{a} = \vec{a_t} + \vec{a_n}$; $a^2 = a_t^2 + a_n^2$ (2)

$a_t = \frac{\Delta v}{\Delta t}$; $v = R\omega$; iz $\omega = const$, $R = const$

$v = const \Rightarrow \boxed{a_t = 0}$ (2)

iii $a_t = R\alpha$; $\alpha = 0 \Rightarrow \boxed{a_t = 0}$

$a_n = \frac{v^2}{R} = \frac{(R\omega)^2}{R} = R\omega^2$

$a_n = 331{,}8 \cdot 10^{-4}\frac{m}{s^2}$ (4)

$a_n = 0{,}0332\frac{m}{s^2}$

2.

a) $\omega = const$

$r = 10\,cm = 0{,}1\,m$

$\theta = 270°$

$t = 1\,s$

$\omega = ?$

$\omega = const \Rightarrow \omega = \frac{\theta_r}{t}$

$2\pi\,rad \equiv 360°$; $\theta_r = 270°$ $\Rightarrow \frac{\theta_r}{2\pi\,rad} = \frac{270°}{360°} = \frac{3}{4}$

$\theta_r = \frac{3\pi}{2}\,rad$ ③

$\omega = \frac{\theta_r}{t}$; $\omega = \frac{\frac{3\pi}{2}\,rad}{1\,s}$ $\Rightarrow$ $\omega = \frac{3\pi}{2}\,\frac{rad}{s}$ ③

$\omega = 4{,}71\,\frac{1}{s}$

b) $t = 1\,min = 60\,s$

$n = ?$

$T = \frac{t}{n}$; $\omega = \frac{2\pi}{T} = \frac{2\pi}{\frac{t}{n}} = \frac{2\pi n}{t}$; $\omega = \frac{\theta_r}{t}$

$\theta_r = \omega t$

$n = \frac{\omega t}{2\pi}$

$\theta_r = \frac{3\pi}{2}\,\frac{rad}{s} \cdot 60\,s$

$\theta_r = 90\pi\,rad$

$n = \frac{\frac{3\pi}{2}\,rad \cdot 60\,s}{2 \cdot 3 \cdot 2\pi} = \frac{180}{4}$

$[n] = \frac{\theta_r}{2\pi}$

$n = 45$ ⑥

ou bien

$[n] = \frac{360°}{2\pi} = 45$

c) $\alpha = ?$ 3

$a = ?$ $\omega = const$; $\left(\omega = \frac{3\pi}{2}\ \frac{rad}{s}\right)$

$\alpha = \frac{\Delta\omega}{\Delta t} = 0$ $\Delta\omega = 0$

$\boxed{\alpha = 0}$ (5)

$\vec{a} = \vec{a_t} + \vec{a_n}$; $a^2 = a_t^2 + a_n^2$ (2)

$a_t = \frac{\Delta v}{\Delta t}$; iz $v = r\omega \Rightarrow v = const \Rightarrow \Delta v = 0$

$\boxed{a_t = 0}$

ili $a_t = r\alpha \Rightarrow \boxed{a_t = 0}$ (2)

$a_n = \frac{v^2}{r}$; $v = r\omega$

$a_n = \frac{(r\omega)^2}{r} = r\omega^2$

$a_n = 0{,}1m \cdot \left(\frac{3\pi}{2}\ \frac{1}{s}\right)^2 = 0{,}1m\ \frac{9\pi^2}{4}\ \frac{1}{s^2}$

$\boxed{a_n = 2{,}22\ \frac{m}{s^2}}$ (4)

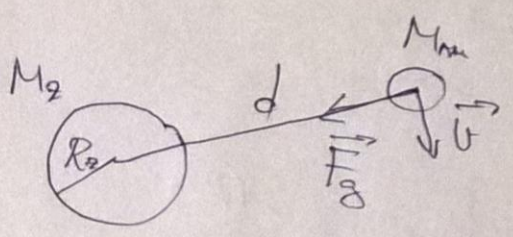
4

$v = 1\,km/s = 10^3 \frac{m}{s}$

$d = 60R$

$R = 6370\,km = 6{,}37 \cdot 10^6\,m$

$M_z = ?$



$\vec{F} = m\vec{a}$ (2)

$\vec{a} = \vec{a_t} + \vec{a_n}$ (2); $v = const \Rightarrow a_t = \frac{\Delta v}{\Delta t} = 0$

$\vec{a} = \vec{a_n}$; (4) $\vec{F} = \vec{F_g}$

$\vec{F_g} = m\vec{a_n}$; $a_n = \frac{v^2}{d}$ (2); $F_g = \gamma \frac{M_z M_m}{d^2}$ (4)

$F_g = m a_n = m\frac{v^2}{d}$

$\gamma \frac{M_z M_m}{d^2} = M_m \frac{v^2}{d} \Rightarrow M_z = \frac{d v^2}{\gamma}$ (8)

$M_z = \frac{60 \cdot 6{,}37 \cdot 10^6\,m \cdot \left(10^3 \frac{m}{s}\right)^2}{6{,}67 \cdot 10^{-11} \frac{Nm^2}{kg^2}}$

$\boxed{M_z \approx 5{,}7 \cdot 10^{24}\,kg}$ (3)

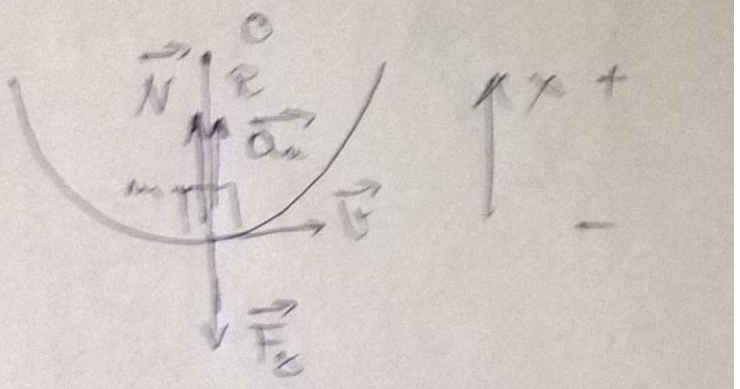
$m = 5t = 5000\,kg$

$v = 60\,km/h = 60 \cdot \frac{1000\,m}{3600\,s} = 16{,}67\,\frac{m}{s}$

$R = 100\,m$

$g = 10\,\frac{m}{s^2}$

$Q = ?$



$\vec{F} = m\vec{a}$ (1)

$\vec{a} = \vec{a}_t + \vec{a}_n$ (2)

$v = const. \Rightarrow \Delta v = 0 \Rightarrow a_t = \frac{\Delta v}{\Delta t} = 0$

$\vec{a} = \vec{a}_n$ (4) $\qquad a_n = \frac{v^2}{R}$ (2)

$\vec{F}_g + \vec{N} = m\vec{a}_n$ (3) $\qquad N = Q$; $F_g = mg$ (2)

$N - F_g = ma_n$ (3)

$N = m(a_n + g)$

$Q = m\left(\frac{v^2}{R} + g\right)$ (4)

$Q = 5000\,kg \left(\frac{(16{,}67\,\frac{m}{s})^2}{100\,m} + 10\,\frac{m}{s^2}\right)$

$Q = 63{,}9 \cdot 10^3\,N$ (3)

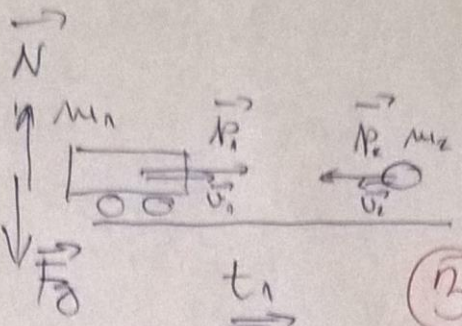
5. $m_1 = 10\,kg$

$v_1 = 1\,m/s$

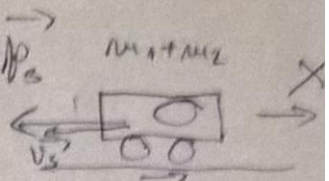
$m_2 = 2\,kg$

$v_2 = 7\,m/s$

$v_3 = ?$



t_1 $\vec{p}_1$ (3)



t_3 $\vec{p}_3$

$\vec{p}_1 = \vec{p}_3$ (5)

$\vec{p}_0 = \vec{p}_1 + \vec{p}_2$

$\vec{p}_3 = \vec{p}_3$

$\Rightarrow$ $\vec{p}_1 + \vec{p}_2 = \vec{p}_3$,

Kema sila duž x-ose pa važi zakon održanja impulsa duž te ose. (2)

$p_3 = |p_1 - p_2|$, pravac isti, a smer većeg vektora

$p_1 = m_1 v_1$ (2) $\quad p_2 = m_2 v_2$ (2)

$p_1 = 10\,kg\frac{m}{s}$ $\quad p_2 = 14\,kg\frac{m}{s}$ $\Rightarrow$ $p_2 > p_1$ $\Rightarrow$ (2)

$\vec{p}_3$ ima smer vektora $\vec{p}_2$

$p_3 = (m_1 + m_2) v_3 = p_2 - p_1$

$v_3 = \frac{p_2 - p_1}{m_1 + m_2}$ (6)

$v_3 = \frac{4\,kg\frac{m}{s}}{12\,kg} = \frac{1}{3}\frac{m}{s}$

$\boxed{v_3 = 0{,}33\frac{m}{s}}$ (3)

6. $m_1 = 20kg$

$v_1 = 50 \frac{m}{s}$

$m_2 = 10t = 10000kg$

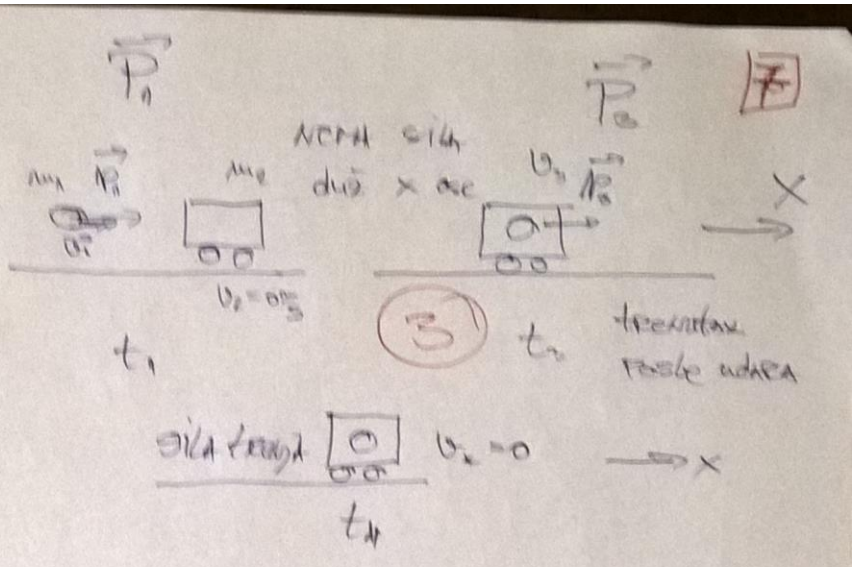
$v_2 = 0 \frac{m}{s}$

$a = 50cm = 0,5m$

$v_k = 0 \frac{m}{s}$

$v_3 = ?$

$\mu = ?$



Primjenimo zakon očuvanja impulsa duž x osi do trenutka sudara (uključujući i sudar) (2)

$\vec{P}_0 = \vec{P}_3$; $\vec{P}_1 = \vec{p}_1 + \vec{p}_2$; $\vec{P}_3 = \vec{p}_3$ (5)

$\vec{p}_1 + \vec{p}_2 = \vec{p}_3$; $p_1 = m_1 v_1$ (2.1)

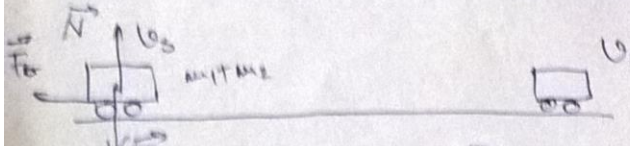
$\vec{p}_1 = \vec{p}_3$ $p_2 = m_2 v_2 = 0$ (2)

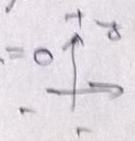
$p_1 = p_3$ $p_3 = (m_1 + m_2) v_3$

$m_1 v_1 = (m_1 + m_2) v_3$ (2)

$v_3 = \frac{m_1 v_1}{m_1 + m_2} \Rightarrow v_3 = \frac{20kg \cdot 50 \frac{m}{s}}{10020kg} = 0,1 \frac{m}{s}$; $\boxed{v_3 = 0,1 \frac{m}{s}}$

NA drugi dio kretanja primjenjujemo II Njutnov zakon





$\vec{F} = (m_1 + m_2)\vec{a}$

$\vec{F}_{tr} + \vec{N} + \vec{F}_g = (m_1 + m_2)\vec{a}$ (4)

$m = m_1 + m_2$

y: $N - F_g = 0 \Rightarrow N = F_g = mg$; $F_{tr} = \mu N$

x: $-F_{tr} = (m_1 + m_2) a$

$-\mu mg = (m_1 + m_2) a \Rightarrow a = const.$ i $a < 0$

Kretanje tijela je usporeno

$v^2 = v_0^2 - 2as$

$v_k^2 = v_3^2 - 2as$

$2as = v_3^2 \Rightarrow |a| = \frac{v_3^2}{2s}$

$|a| = \frac{(0{,}1\ m/s)^2}{2 \cdot 0{,}5\ m} = 0{,}01\ \frac{m}{s^2}$

$a = -0{,}01\ \frac{m}{s^2}$ (2)

$-\mu(m_1+m_2)g = (m_1+m_2)a$

$\mu = -\frac{a}{g}$

$\mu = -\frac{-0{,}01\ m/s^2}{10\ m/s^2} \Rightarrow \boxed{\mu = 0{,}001}$ (2)

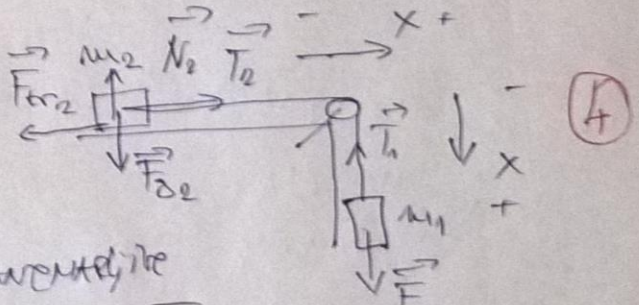
Ž.

$m_1 = 250\ g = 0{,}25\ kg$

$m_2 = 400\ g = 0{,}4\ kg$

$\mu = 0{,}03$

$a = ?\ ,\ T = ?$



(4)

Niti su neistegljive i zanemarljive mase pa i kotur $\Rightarrow T_1 = T_2 = T$; (2)

$a_1 = a_2 = a$

II Njutnov zakon primijenjen na 1 tijelo

$\vec{F}_1 = m_1\vec{a}_1$ (3)

$\vec{F}_{g1} + \vec{T}_1 = m_1\vec{a}_1$

x: $F_{g1} - T_1 = m_1 a_1 \Rightarrow \boxed{m_1 g - T = m_1 a}$ (3)

$F_{g1} = m_1 g$

2. тіло 2

$$\vec{F}_2 = m_2\vec{a}_2$$

$$\vec{F}_{g_2} + \vec{T}_2 + \vec{N}_2 + \vec{F}_{тр_2} = m_2\vec{a}_2 \quad (2)$$

x: $T_2 - F_{тр_2} = m_2 a_2 \Rightarrow T - F_{тр_2} = m_2 a$ (2')

$F_{тр_2} = \mu N_2$

y: $\vec{N}_2 + \vec{F}_{g_2} = \vec{0} \Rightarrow N_2 - F_{g_2} = 0 \Rightarrow N_2 = F_{g_2} = m_2 g$ (2')

$F_{тр_2} = \mu \cdot m_2 g$

(1) $\boxed{T - \mu m_2 g = m_2 a}$

(2) $\boxed{m_1 g - T = m_1 a}$ $+$

$\Rightarrow m_1 g - \mu m_2 g = a(m_1 + m_2)$

$g(m_1 - \mu m_2) = a(m_1 + m_2)$

$a = \frac{m_1 - \mu m_2}{m_1 + m_2} g$ (4)

$\Rightarrow a = \frac{0{,}25\,кг - 0{,}05 \cdot 0{,}4\,кг}{0{,}65\,кг} \cdot 10\frac{м}{с^2}$

$\boxed{a = 3{,}66\frac{м}{с^2}}$

із 2. знайдемо інше $T = m_1 g - m_1 a = m_1(g - a)$ (3)

$T = 0{,}25\,кг\,(10\frac{м}{с^2} - 3{,}66\frac{м}{с^2})$

$\boxed{T = 1{,}585\,Н}$

8.

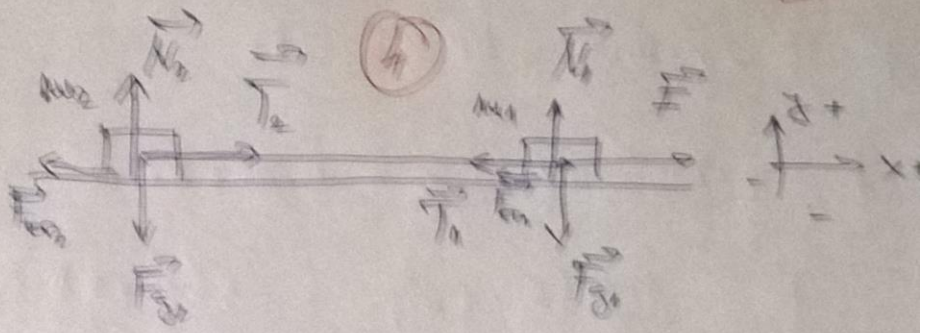
$m_1 = 0{,}2\,kg$

$m_2 = 0{,}3\,kg$

$F = 10\,N$

$\mu = 0{,}1$

$a = ?$



① ②

Niti neistegljiva i zanemarive mase $\Rightarrow$

$T_1 = T_2 = T$ i $a_1 = a_2 = a$ ②

II Njutnov zakon 1. tijelo $\vec{F}_{e_1} = m_1 \vec{a}_1$

$\vec{F} + \vec{N}_1 + \vec{T}_1 + \vec{F}_{tr_1} + \vec{F}_{g_1} = m_1 \vec{a}_1$ ②

x osa: $\vec{F} + \vec{T}_1 + \vec{F}_{tr} = m_1 \vec{a}_1$ ②

$F - T_1 - F_{tr_1} = m_1 a_1 \Rightarrow \underline{F - T - F_{tr_1} = m_1 a}$

y osa: $\vec{N}_1 + \vec{F}_{g_1} = \vec{0} \Rightarrow N_1 - F_{g_1} = 0 \Rightarrow N_1 = F_{g_1} = m_1 g$

$F_{tr_1} = \mu N_1 = \mu m_1 g$ ②

$\boxed{F - T - \mu m_1 g = m_1 a}$ ①

2. tijelo $\vec{F}_{e_2} = m_2 \vec{a}_2$

$\vec{T}_2 + \vec{N}_2 + \vec{F}_{tr_2} + \vec{F}_{g_2} = m_2 \vec{a}_2$ ②

x osa: $\vec{T}_2 + \vec{F}_{tr_2} = m_2 \vec{a}_2$

$T_2 - F_{tr_2} = m_2 a_2 \Rightarrow \underline{T - F_{tr_2} = m_2 a}$ ②

y osa: $\vec{N}_2 + \vec{F}_{g_2} = \vec{0}$

$N_2 - F_{g_2} = 0 \Rightarrow N_2 = F_{g_2} = m_2 g$ ②

$F_{tr_2} = \mu \cdot N_2 = \mu m_2 g$

$T - \mu m_2 g = m_2 a$ (2)

$F - T - \mu m_1 g = m_1 a$ (1)

+

(1)

$F - \mu m_2 g - \mu m_1 g = m_1 a + m_2 a$

$F - \mu g (m_1 + m_2) = a (m_1 + m_2)$

(7)

$$a = \frac{F - \mu g (m_1 + m_2)}{m_1 + m_2} = \frac{F}{m_1 + m_2} - \mu g$$

$a = 20 \frac{m}{s^2} - 1 \frac{m}{s^2}$

$a = 19 \frac{m}{s^2}$