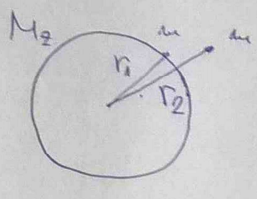
Njutnov zakon gravitacije

224.

$\frac{F_{g2}}{F_{g1}} = \frac{1}{100}$

$h_1 = 0m$



$r_1 = R_z = 6400\,km = 6{,}4 \cdot 10^6\,m$

$r_2 = ?$ $\quad h_2 = ?$

$r_1 = R_z + h_1 = R_z$

$r_2 = R_z + h_2$

$F_{g1} = \gamma \frac{M_z \cdot m}{r_1^2} = \gamma \frac{M_z \cdot m}{R_z^2}$

$F_{g2} = \gamma \frac{M_z \cdot m}{r_2^2} = \gamma \frac{M_z \cdot m}{(R_z + h_2)^2}$

$\frac{F_{g2}}{F_{g1}} = \frac{\gamma \frac{M_z \cdot m}{(R_z + h_2)^2}}{\gamma \frac{M_z \cdot m}{R_z^2}} = \frac{R_z^2}{(R_z + h_2)^2} = \left(\frac{R_z}{R_z + h_2}\right)^2$

$\frac{F_{g2}}{F_{g1}} = \frac{1}{100} \Rightarrow \frac{1}{100} = \left(\frac{R_z}{R_z + h_2}\right)^2$ | korjenujemo jednačinu

$\sqrt{\frac{1}{100}} = \sqrt{\left(\frac{R_z}{R_z + h_2}\right)^2}$ { $\sqrt{\frac{1}{100}} = \frac{\sqrt{1}}{\sqrt{100}} = \frac{1}{10}$

$\frac{1}{10} = \frac{R_z}{R_z + h_2} \Rightarrow R_z + h_2 = 10 R_z$

$\boxed{h_2 = 9R_z}$

$h_2 = 9 \cdot 6{,}4 \cdot 10^6\,m$

$h_2 = 57{,}6 \cdot 10^6\,m$

27.

$r = 3{,}8 \cdot 10^8 m$

$T = 27{,}3 \text{ dana} = 27{,}3 \cdot 24\,h = 27{,}3 \cdot 24 \cdot 3600 s$

$M_m = 7{,}3 \cdot 10^{22} kg$

$F_{cp_m} = ?$ $F_g = ?$

$F_{cp} = m a_m = M_m \cdot a_m$

$a_m = \frac{v^2}{r}$; $v = \frac{s}{t} = \frac{2r\pi}{T}$

$a_m = \frac{\left(\frac{2r\pi}{T}\right)^2}{r} = \frac{4r^2\pi^2}{r T^2} = \frac{4r\pi^2}{T^2}$

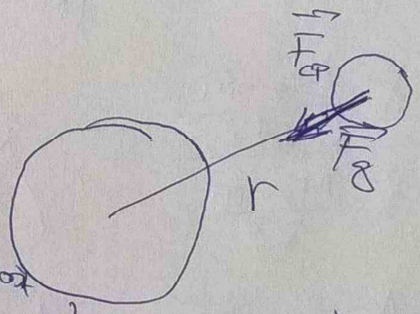
ili

$a_m = \frac{v^2}{r}$; $v = r\omega$

$\omega = \frac{2\pi}{T} \Rightarrow v = \frac{r 2\pi}{T}$

$a_m = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2}$

$$\boxed{F_{cp} = M_m \cdot \frac{4r\pi^2}{T^2}}$$



Jedina sila koja djeluje na tijelo je gravitaciona koja djeluje duž poluprečnika putanje pa je ona ustvari centripetalna sila.

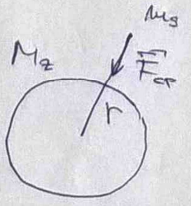
$F_g = F_{cp} = M_m \frac{4\pi^2 r}{T^2}$

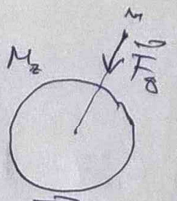
228.

a) $M_z = 6 \cdot 10^{24} kg$; $R_z = 6{,}4 \cdot 10^6 m$

$r = 7000 km = 7 \cdot 10^6 m$

$a_{cp} = ?$ $\quad v = ?$





$$a_{cp} = a_n = \frac{v^2}{r}$$

$\vec{F}_{cp}$ je $\vec{F}_g$; $\vec{F}_{cp} = \vec{F}_g$

II Kretanje satelita

$\vec{F} = m_s \vec{a}$; $\vec{F}_g = m_s \vec{a}_n$

konstantan intenz. sile $\Rightarrow v = const.$

$\Rightarrow a_t = 0$

$\vec{F}_g = m_s \vec{a}_n \Rightarrow F_g = m_s a_n$

$$F_g = \gamma \frac{M_z m_s}{r^2} \Rightarrow \gamma \frac{M_z m_s}{r^2} = m_s \cdot a_n$$

$$\boxed{a_n = \gamma \frac{M_z}{r^2}}$$

$$a_n = \frac{v^2}{r} \Rightarrow v^2 = a_n r \Rightarrow \boxed{v = \sqrt{a_n r}}$$

b) $h = 3200 km$

$v = ?$ $\quad \vec{F} = m\vec{a}$

$r = R_z + h$

$F_g = m_s a_n$; $F_g = \gamma \frac{M_z \cdot m_s}{r^2} = \gamma \frac{M_z \cdot m_s}{(R_z+h)^2}$

$a_n = \frac{v^2}{r} = \frac{v^2}{R_z + h}$

$$\gamma \frac{M_z \cdot m_s}{(R_z+h)^2} = m_s \cdot \frac{v^2}{(R_z+h)}$$

$$v^2 = \gamma \frac{M_z}{R_z + h} \Rightarrow v = \sqrt{\gamma \frac{M_z}{R_z + h}}$$

$$v = \sqrt{6{,}67 \cdot 10^{-11} \frac{N m^2}{kg^2} \; \frac{6 \cdot 10^{24} kg}{6{,}4 \cdot 10^6 m + 3{,}2 \cdot 10^6 m}}$$

$$v = \sqrt{6{,}67 \cdot 10^{-11} \frac{kg \cdot \frac{m}{s^2} \cdot m^2}{kg^2} \; \frac{6 \cdot 10^{24} kg}{10^6 m (6{,}4 + 3{,}2)}}$$

$$v = \sqrt{\frac{6{,}67 \cdot 6}{9{,}6} \cdot 10^{-11+24-6} \frac{m^2}{s^2}}$$

$$v = \sqrt{\frac{6{,}67 \cdot 6}{9{,}6} \cdot 10^{7} \frac{m^2}{s^2}} = \sqrt{\frac{6{,}67 \cdot 6}{9{,}6} \cdot 10 \cdot 10^{6} \frac{m^2}{s^2}} =$$

$$= \sqrt{\frac{66{,}7 \cdot 6}{9{,}6}} \cdot 10^3 \frac{m}{s} \qquad \sqrt[2]{10^6} = 10^{\frac{6}{2}} = 10^3$$

$$\boxed{v = \sqrt{\frac{66{,}7 \cdot 6}{9{,}6}} \cdot 10^3 \frac{m}{s}}$$

229.

a) $r = 9500\,km$

$M_m = 6 \cdot 10^{23}\,kg$

$v = ?$, $T = ?$

$\vec{F} = m\vec{a}$

$F_g = m_t a_n$

$\gamma \frac{M_m \cancel{m_t}}{r^{\cancel{2}}} = \cancel{m_t} \cdot \frac{v^2}{\cancel{r}}$

$v^2 = \gamma \frac{M_m}{r} \Rightarrow \boxed{v = \sqrt{\gamma \frac{M_m}{r}}}$

$F = F_g = \gamma \frac{M_m \cdot m_t}{r^2} = \text{co}$

$\Rightarrow a_t = 0$

$a = a_n$

$a_n = \frac{v^2}{r}$

b) $v = 7{,}8 \frac{km}{s} = 7{,}8 \cdot 10^3 \frac{m}{s}$

$r = ?$

$t = 1\,\text{day}$, $M_z = 6 \cdot 10^{24}\,kg$; $1\,\text{day} = 24 \cdot 3600\,s$

$n = ?$

$v = 7{,}8 \cdot 10^3 \frac{m}{s} = \text{const} \Rightarrow a_t = 0 \Rightarrow a = a_n$

$\vec{F} = m\vec{a} \Rightarrow F = ma$

$F = F_g$; $a = a_n = \frac{v^2}{r}$; $F_g = \gamma \frac{M_z \cdot m}{r^2}$

$F_g = m a_n = m \frac{v^2}{r}$

$\gamma \frac{M_z \cancel{m}}{r^{\cancel{2}}} = \cancel{m} \frac{v^2}{\cancel{r}} \Rightarrow \boxed{r = \gamma \frac{M_z}{v^2}}$

$v = \frac{s}{t}$; $s = n \cdot 2r\pi \Rightarrow v = \frac{n \cdot 2r\pi}{t}$

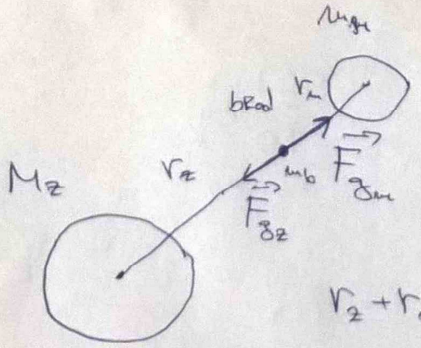
$\boxed{n = \frac{v \cdot t}{2r\pi}}$

230. $F_{gz} = 0N$

$\frac{M_z}{m_m} = 81$

$r_{zm} = 3{,}8 \cdot 10^8 m$

$r_z = ?$



$r_z + r_m = r_{zm}$

$\vec{F}_{gz} = m_b \vec{a}$

II Njutnov zakon za brod

$\vec{F}_{gz} = \vec{F}_{gz} + \vec{F}_{gm}$ – sabiranje vektora istih pravaca i suprotnih smerova

$F_{gz} = |F_{gz} - F_{gm}| = 0N$

$F_{gz} = F_{gm}$

$F_{gz} = \gamma \frac{M_z \cdot m_b}{r_z^2}$

$F_{gm} = \gamma \frac{m_m \cdot m_b}{r_m^2}$

$\gamma \frac{M_z m_b}{r_z^2} = \gamma \frac{m_m m_b}{r_m^2}$

$M_z r_m^2 = m_m r_z^2 \quad / : m_m$

$\frac{M_z}{m_m} r_m^2 = r_z^2$

$81 r_m^2 = r_z^2$ / korijenujemo izraz

$\sqrt{81 r_m^2} = \sqrt{r_z^2}$

$\begin{cases} 9 r_m = r_z \\ r_m = r_{zm} - r_z \end{cases}$ i $r_z + r_m = r_{zm} \Rightarrow$

$9(r_{zm} - r_z) = r_z$

$9 r_{zm} - 9 r_z = r_z$

$9 r_{zm} = 10 r_z$

$r_z = \frac{9}{10} r_{zm}$